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Hologravity

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Abstract: The dS/dS correspondence provides a holographic description of quantum gravity in d dimensional de Sitter space near the horizon of a causal region in a well defined approximation scheme; it is equivalent to the low energy limit of conformal field theory on de Sitter space in $d-1$ dimensions coupled to $d-1$ dimensional gravity. In this work, we extend the duality to higher energy scales by performing calculations of various basic physical quantities sensitive to the UV region of the geometry near the center of the causal patch. In the regime of energies below the d dimensional Planck scale but above the curvature scale of the geometry, these calculations encode the physics of the $d-1$ dimensional matter plus gravity system above the crossover scale where gravitational effects become strong. They exhibit phenomena familiar from studies of two dimensional gravity coupled to conformal field theory, including the cancellation of the total Weyl anomaly in $d-1$ dimensions. We also outline how the correspondence can be used to address the issue of observables in de Sitter space, and generalize the correspondence to other space times, such as black holes, inflationary universes, and landscape bubble decays. In the cases with changing cosmological constant, we obtain a dual description in terms of renormalization group flow.

Keywords: AdS-CFT, dS-CFT Correspondence.

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Contents

1. Introduction

In [1] we established a holographic duality between gravity on d-dimensional de Sitter space (dS_d) and a CFT on dS_{d-1} below the scale of the inverse curvature radius $1/L$, coupled to $d-1$ dimensional gravity. In this paper, we extend the duality above the scale $1/L$, by a set of controlled computations in the bulk d-dimensional theory. These computations sum up effects of the residual gravity coupled to matter in $d-1$ dimensions. We also describe qualitatively a formulation of the system based on further holographic reduction, and a dual description of backgrounds with changing cosmological constant (as in inflation and landscape decays) in terms of renormalization group flow.

The basic observation in [1] was that when written in a dS_{d-1} slicing, dS_d has the form of a Randall-Sundrum system [2] with a smooth, built in analogue of the UV brane cutting off the theory in the UV. The metric for the dS_d static patch can be written

$$ds^2_{dS_d} = \frac{1}{\cosh^2\left(\frac{z}{L}\right)}(ds^2_{dS_{d-1}} + dz^2)\,. \tag{1.1}$$

Close to the horizon the warped geometry of dS_d is isomorphic to the dS_{d-1} slicing of AdS_d (for which the $1/\cosh^2(z/L)$ in the warped metric (1.1) is replaced by $1/\sinh^2(z/L)$). For latter one has a holographic duality in terms of a conformal field theory on dS_{d-1}. Hence given the UV/IR correspondence in the AdS/CFT dictionary, we can conclude that at low energies, the dS causal patch is equivalent to a CFT on dS_{d-1}.

In AdS_{d-1}, the warp factor diverges towards the UV region of the geometry (far away from the horizon) and $d-1$ dimensional gravity decouples. In our case, as in Randall-Sundrum, the warp factor is bounded in the solution and one finds a dynamical $d-1$ dimensional graviton. In the Randall-Sundrum construction, one truncates the warp factor at a finite value of the radial coordinate by including a brane source (or a compactification manifold) with extra degrees of freedom. In the dS case, the additional brane source is unnecessary; a smooth UV brane at which the warp factor turns around is built in to the geometry [3].

In this paper, we make basic computations determining how the lower dimensional theory behaves above the scale $1/L$. Our d dimensional gravity calculations provide a controlled formulation of the system which translates in the $d-1$ dimensional language (via a simple conformal map relating our system to one with AdS asymptotics) to an effective description in which the total (matter plus gravity) central charge is zero and the operator dimensions are reduced from their field theoretic values. These results are reminiscent of features familiar in two dimensional gravity — namely gravitational dressing of operators and absence of a conformal anomaly including the effects of gravity. While these qualitative conclusions are to be expected, the values of the resulting operator dimensions remain somewhat mysterious, as we will discuss further after explaining the computations. In general, our results sum up the combined effects of $d-1$ dimensional gravity, matter, and their interactions between the scale $1/L$ and the bulk Planck scale M_d.

Some of our results here also apply to the Randall-Sundrum system, providing a bulk computation of some of the effects of induced gravity in that more general context. However, the dS case appears especially simple — in particular the absence of an external brane source gives rise to the possibility that in the case of the dS/dS correspondence, unlike in RS, no new degrees of freedom have to be introduced in the dual conformal field theory at the scale $1/L$. The interplay of the two conformal field theories with the localized graviton may completely fix the dual and naturally give rise to the physics of the crossover scale $1/L$, though further computations would be required to test this.

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After repeated application of the same duality down to 1+1 or 0+1 dimensional gravity, the holographic dual is a well defined, UV complete theory. Basically it boils down to a standard quantum mechanical system with a hamiltonian constraint. As a result, the observables of the full de Sitter space are similar to those in toy models of quantum gravity obtained by truncation to minisuperspace and reduction to one dimensional quantum mechanics, albeit with a much richer matter sector.

Finally, we show that decays in cosmological constant correspond to a time dependent RG flow in our setup, similarly to [4] but here in a system with a Lorentzian-signature CFT sector. In particular, in our setup the time direction is common in both the d and $d-1$ descriptions, so it is particularly straightforward to incorporate time dependent physics. The bulk gravitational physics corresponding to the region near the horizon translates into physics of the matter sector, in this case yielding a relation between inflation and a time dependent RG flow.

In the following section we will review in more detail the scales that are involved in the dS/dS correspondence. We will also examine repeated applications to 1+1 or 0+1 dimension, allowing one to define dS quantum gravity via the observables of a gravity+matter system in one dimension. Section 3 then describes our method to extract field theory information from the bulk in general dimensionality. By mapping standard dS physics to a system with position dependent masses and couplings in AdS, we can apply the well established AdS/CFT dictionary to obtain the $d-1$ dimensional description of various UV quantities of interest in the dS/dS correspondence. In section 4 we present results for the heat capacity of the dual FT and the conformal anomaly. In section 5 we extend the dS/dS correspondence to a Black Hole/Black Hole correspondence and find that the geometry is consistent with the properties of the field theory we uncovered before, which we then summarize in section 6. A discussion for time dependent backgrounds with changing cosmological constant follows in section 7. In section 8 we present our conclusions.

2. Properties of the dS/dS correspondence

2.1 The scales of interest

As reviewed above, the $d-1$ dimensional holographic dual of dS_d is only a pair of CFTs up to the energy scale $1/L$. On the d dimensional gravity side of the correspondence, one has a local effective field theory description good up to the d-dimensional Planck scale $M_d \gg 1/L$ (or perhaps the bulk string scale in a stringy construction). At scales above M_d quantum gravity effects become important in the bulk and one has to appropriately UV complete the system, for example by embedding it as a metastable dS into string theory following one of the constructions [5−9]. Most of our analysis in this paper will be concerned with using the gravity side of the correspondence to determine the behavior of the $d-1$ theory in the range of energies $1/L < E < M_d$.

In the $d-1$ description, the Planck mass is dominated by an induced contribution, of order $M_{d-1}^{d-3} \sim S/L^{d-3}$ where $S \sim (M_d L)^{d-2} \sim (M_{d-1} L)^{d-3}$ is the Gibbons-Hawking entropy of dS_d and the effective species number (central charge) of our dual low energy CFTs.

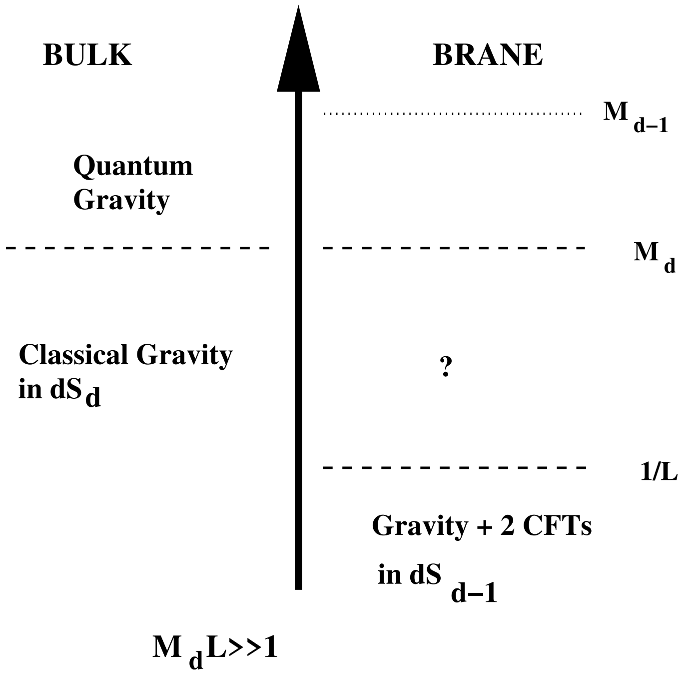

Figure 1: The hierarchy of scales. M_{d-1}^{d-3} =L M_{d}^{d-2} appears as an induced scale beyond M_d, the ultimate cutoff of the theory.

These scales of interest are summarized in figure 1.

Many of the calculations we perform in the next section are aimed at exploring the lower dimensional field theory plus gravity physics in the range of energies $1/L < E < M_d$ by using bulk gravity. The effective action in the $d-1$ description is of the form

$$S_{\text{eff}} = M_{d-1}^{d-3} \int d^{d-1}x \left(\mathcal{R} + L^2\mathcal{R}^2 + L^4\mathcal{R}^4 + \cdots\right) + \int d^{d-1}x \mathcal{L}_{\text{matter}} \,. \tag{2.1}$$

In particular, there are extra terms in the gravitational action which arise from loops of the S matter species and are only suppressed by inverse powers of $1/L$. In this sense, gravity becomes strong at the scale $1/L$. However, the interactions of matter with gravity are still suppressed; the field equations following from the above action are of the form

$$\mathcal{R} + L^2\mathcal{R}^2 + L^4\mathcal{R}^4 + \cdots \sim \frac{1}{M_{d-1}^{d-3}} T_{\text{matter}} \tag{2.2}$$

and so for matter energy scales E satisfying

$$E^{d-1} \ll \frac{S}{L^{d-1}} \tag{2.3}$$

the higher curvature terms in (2.1), (2.2) are negligible.

The d-dimensional description makes clear that a local description of the physics is available all the way up to the scale M_d, and in the regime (2.3) the interactions of matter are not strongly corrected by gravity. Correspondingly, one does not find $d-1$ dimensional black holes in the range $1/L < E < M_d$.

Nonetheless, we will find that the local description in this range of energies $1/L < E < M_d$ is softened in ways suggesting a role for gravity. Namely, in a precise sense we will specify, the effective operator dimensions and the effective central charge decrease dramatically for $E \gg 1/L$. These effects (and the other related effects we compute in what follows) suggest gravitational dressing and cancellation of the conformal anomaly in the $d-1$ theory at energies above $1/L$.

In addition, we may have couplings in the matter sector of (2.1) among operators of the two low energy CFTs at or above the scale $1/L$. We will see from a generalization of the calculations in [10, 11] that such interactions must be present.

We will revisit the interpretation of the physics in the range $1/L < E < M_d$ after computing the behavior of various basic physical quantities in this regime.

2.2 de Sitter Quantum Gravity

For general d both sides of the duality contain quantum gravity and hence need to be UV completed above the scale M_d. Repeated application of the same duality can map dS_d gravity in any d down to 2+1, 1+1 or 0+1 dimensions, where gravity becomes non-dynamical. In particular when reducing the system down to two or one dimension, the lower dimensional gravitational physics is a UV complete theory even above scales M_d. Therefore once we established that the two theories are equivalent up to scales M_d, one can take the lower dimensional system as one possible UV completion of bulk gravity! The full description of the system then is this low dimensional gravitational system coupled to a complicated matter sector.

For concreteness let us focus to the scenario where we dualize all the way down to 0+1 dimensions, but a similar case could be made for 1+1 as well. What we end up with is a standard quantum mechanical theory with hamiltonian and Hilbert space coupled to 1d gravity on dS_1. What does this mean? Since the only space-time dimension we are left with is time, the metric is trivial. The only sense in which we are on dS_1 is that we are instructed to study the theory at a finite temperature $T = \frac{1}{2\pi L}$. What does it mean to couple the system to gravity? In 0+1 dimensions gravity doesn't have any dynamical degrees of freedom. It does, however, impose a constraint

$$H = \mathrm{const} \tag{2.4}$$

on the hamiltonian as a consequence of time reparametrization invariance. Like for the 2d string worldsheet this constraint can be imposed on the level of states.

Another consequence of (2.4) is that there is no interesting time evolution of the physical states. The wavefunction of the universe is fixed, there are no outside observers that could measure the wavefunction of the universe. The best known example of such a quantum mechanical system with hamiltonian constraint due to time reparametrization invariance is the worldline of a relativistic point particle. For dS/dS one needs simply to replace the $(\dot{X})^2$ term in the point particle action with a more general theory involving many degrees of freedom.

Then what are the observables in such a constrained quantum mechanical system? Since there is no non-trivial time evolution, the usual scattering amplitudes will not be interesting, but one can define conditional probabilities based on chains of projection operators and decoherence functionals. This has been addressed for example in [12]. The original motivation for studying reparametrization invariant quantum mechanics as a model for quantum cosmology was derived from Minisuperspace, where the space of all metrics gets truncated to the space of all FRW like universes and the quantum gravity just becomes quantum mechanics of the scale factor. In that case, the truncation was just a toy model; in dS/dS it is the full story derived from holography (given the rich matter sector corresponding to the bulk geometry). This provides a new motivation to develop this formalism as a framework for quantum cosmology.

3. Holographic calculation in de Sitter space via AdS/CFT

In this section, we study the linearized theory of fields — scalars and the graviton — in the d dimensional description. A given bulk mode can play two roles in our system: it sources operators in the two CFTs, and if light enough contains an extra "localized" $d-1$ dimensional mode independent of the degrees of freedom of the two CFTs. We start by reviewing the mode spectrum and applying it to calculate aspects of the communication between the two CFT throats built into dS_d. Then we move on to focus on the physics contained in each throat, by making a conformal map to AdS in a way which allows us to read off the behavior of the effective operator dimensions and central charge of the system in the deep UV ($E \gg 1/L$).

3.1 Modes in dS

In [1] we reviewed the mode spectrum in $d-1$ dimensions descending from the dS_d causal patch sliced by dS_{d-1} slices (1.1). At large z/L, there is a continuum of modes corresponding to the CFT degrees of freedom. In addition, there can be extra bound states in the potential supported near $z=0$ such as the localized graviton [3]. In this subsection we will determine the spectrum of such modes in the case of bulk scalar fields of mass M and comment on its interpretation in the dual.

The mass m of the zero mode will depend on the bulk mass M of the scalar field X. In order to determine this dependence, we need to solve the analog quantum mechanics for a massive scalar field.

As discussed in [1], scalar fields of mass M in dS_d satisfy an equation of motion isomorphic to a Schrödinger equation with potential

$$V = \frac{(d-2)^2}{4L^2} - \left(\frac{(d-1)^2-1}{4L^2} - M^2\right)\frac{1}{\cosh^2(z)}\,. \tag{3.1}$$

Here we separate variables as in [1, 3] and denote by m the mass of the $d-1$ particle arising from this dimensional reduction.

Analyzing this Schrodinger problem using a mapping to an analogue supersymmetric quantum mechanics problem yields the result

$$m^2 = \frac{1}{2L^2}(d-1) - M^2 - \frac{1}{2L^2}\sqrt{(d-1)^2 - 4L^2M^2}\,. \tag{3.2}$$

For $M = 0$ we get a massless zero mode with $m = 0$ as for the graviton. At the conformally coupled value in the bulk, $M^2 = \frac{(d-1)^2-1}{4L^2}$, the boundstate becomes marginal with $m^2 = \frac{(d-2)^2}{4L^2}$, since at this value the $\cosh^2$ term in the potential (3.1) changes sign. For larger values of M we lose the bound state.[1]

The absence of a bound state for sufficiently large masses has interesting implications. For example, consider the case that we have a perturbative string description in the bulk, so that amplitudes soften above the scale m_s, providing a UV completion of gravity in the bulk. In the $d-1$ description, the string mass modes will not survive as $d-1$ particles. Hence the $d-1$ gravity theory amplitudes are not rendered finite by stringy physics, but instead must be UV completed via a different form of quantum gravity. This is reminiscent of relations between perturbative string limits and the 11 dimensional supergravity limit of M theory via strong-weak coupling dualities; the latter is not UV completed by a tower of string states. In our case, for $d = 4$, the holographic dual theory is $2+1$ dimensional gravity, which has been argued to be well defined on its own [14].

3.1.1 Communication between throats

In [10, 11] interactions between low energy field theories corresponding to warped throats were studied via gravity-side calculations. The interactions between operators $\mathcal{O}_1$ corresponding to the low energy effective theory in one throat and $\mathcal{O}_2$ in another were found to be suppressed generically only by powers of the curvature radius scale $1/L$.

In our case, we can perform a similar calculation. Let us denote the $d-1$ dimensional mass of the "glueball" we send across the barrier m. Using the above potential barrier (3.1), the transmission probability across the barrier works out to be

$$T(M', m') = \frac{\sinh^2(\pi m' L)}{\sinh^2(\pi m' L) + \sinh^2 \frac{\pi}{2}\sqrt{4(M'^2L^2 - 1)}} \tag{3.3}$$

where $m'^2 = m^2 - (d-2)^2/4L^2$ and $M'^2 = M^2 - (d^2-2d)/4L^2$.

This formula exhibits the same general behavior noted in [11, 10]: the interactions between the two throats are suppressed only by powers of energy (m) divided by $1/L$. Even after taking into account the enhancement of gravitational interactions due to the large number of species running in the matter loops, it seems this unsuppressed tunneling rate can not be accounted for by gravity alone. We have to add some explicit couplings between the 2 CFTs. Later we will argue that some explicit couplings between the two CFTs are also required just to get bulk fields to be continous across the UV-brane.

[1] The formula for m^2 however still gives a real answer up to bulk masses of $M = \frac{d-1}{2L}$, which is precisely the value of M at which the bulk behavior changes from exponential to oscillatory eigenfunctions (μ becomes imaginary in the language of [13]).

3.2 Conformal mapping from dS to AdS physics

3.2.1 Basic Strategy

In dS slicing the metric of dS_d reads

$$ds^2_{dS_d} = \frac{L^2}{\cosh^2(z)}\left(ds^2_{dS_{d-1}} + dz^2\right) \tag{3.4}$$

while AdS_d can be written as

$$ds^2_{AdS_d} = \frac{L^2}{\sinh^2(z)}\left(ds^2_{dS_{d-1}} + dz^2\right) = \frac{1}{\tanh^2(z)} ds^2_{dS_d} \tag{3.5}$$

so the two are related by a simple conformal transformation. We can use this to map the physics in dS to dynamics in AdS, albeit with unusual actions. Namely the conformal map yields scalars with position dependent masses and gravity with a position dependent Newton constant. By applying the AdS/CFT dictionary to the resulting system, this allows us to make a direct comparison of the UV behavior of the $d-1$ dual of dS_d to the UV behavior of a strongly 't Hooft coupled CFT.

Note that this conformal map takes only half of our dS setup (one of the two throats) to a full AdS slice. So we expect this analysis to capture the effective dynamics of either one of the two CFTs, with the effects of gravity and of the other CFT folded in as they arise above the scale $1/L$. The latter in particular will be encoded in the boundary conditions on AdS. In the dS space fields are continuous, so after the conformal map to AdS we get a theory on two copies of AdS where the boundary value of a given field in one copy sets the boundary conditions for the same field in the second copy, as we will discuss in more detail later. From now on we will mostly set $L = 1$ and only restore L when necessary.

The AdS/CFT dictionary relates bulk scattering amplitudes to local Greens functions in the dual CFT. In our case, gravity is present in the system, and hence local physics is not expected down to arbitrarily short distance scales. Nonetheless, formulating the computations in terms of the AdS/CFT dictionary proves useful: we will find that because of the radially running bulk field masses and Planck mass, short distance physics is softened in a way we can quantify. For example, while we can express bulk amplitudes in terms of a two point function of operators in $d-1$ dimensions, the dimensions of these operators are given by a universal finite value in the UV for each spin of bulk field. The specific heat, a measure of the degrees of freedom, also shuts off in the UV in our system in contrast to AdS/CFT but in a way we can study using the above map to the AdS/CFT dictionary.

3.2.2 Scalar Fields

Let us first consider the case of a scalar field. The bulk action for a free, massive scalar field in dS_d is

$$S = \int d^d x \sqrt{-g}\left(-(\partial_\mu X) - (M^2 + \xi R) X^2\right) \tag{3.6}$$

g_{mn}	$\rightarrow$	$f^2 g_{mn}$
X	$\rightarrow$	$f^{-\frac{d-2}{2}} X$
$\sqrt{-g}$	$\rightarrow$	$f^d \sqrt{-g}$
$-\sqrt{-g}(\partial X)^2$	$\rightarrow$	$-\sqrt{-g}(\partial X)^2 - \sqrt{-g}\frac{(d-2)}{2} X^2 (\nabla^2 \omega) - \sqrt{-g}\frac{(d-2)^2}{4} X^2 (\nabla \omega)^2$
$\sqrt{-g} R$	$\rightarrow$	$f^{d-2}\sqrt{-g}(R - 2(d-1)(\nabla^2\omega) - (d-2)(d-1)(\nabla\omega)^2)$
$-\sqrt{-g}\xi R X^2$	$\rightarrow$	$-\sqrt{-g}\xi R X^2 + 2\sqrt{-g}\xi(d-1)(\nabla^2\omega) + \sqrt{-g}\xi(d-2)(d-1)(\nabla\omega)^2$
$-2\sqrt{-g}\Lambda$	$\rightarrow$	$-2f^d\sqrt{-g}\Lambda$

Table 1: Transformations under conformal rescaling; $\omega = \log f$

where we allowed explicit mass terms as well as mass terms that arise from a coupling to the constant background curvature. Table (1) summarizes the transformation properties of the various terms in the action under a conformal rescaling with a general function f, where $\omega = \log f$. We see that the scalar kinetic term reproduces itself together with some mass like terms. Also the explicit mass terms change. So a scalar of mass M in dS maps again to a scalar field in AdS, but with a different mass M_{total}. Since the conformal factor f depends on the z-coordinate, these are position dependent masses in AdS. In order to determine the dimension of the dual operator one only needs the UV value of that mass, that is the value it takes close to the boundary. Of course the full function $M_{\text{total}}(z)$ will encode interesting information of how the dimensions of operators evolve as we go from the UV to the IR. For our case we need

$$ds^2_{dS_d} = \tanh^2(z) ds^2_{AdS_d}\,, \qquad f = \tanh(z)\,. \tag{3.7}$$

With this we get for the AdS mass:

$$M^2_{\text{total}} = \tanh^2(z)\left(M^2 + \xi d(d-1)\right) - \frac{d(d-2)}{4}\left(1 + \tanh^2(z)\right). \tag{3.8}$$

where we used $R = -d(d-1)$ for the AdS$_d$. In the UV ($z = 0$) the original dS mass term M scales to zero, as does the original ξ term. Instead we get the universal result

$$M^2_{\text{total}} = -\frac{d(d-2)}{4} \qquad \text{for all } \xi,\, M\,. \tag{3.9}$$

That is we get a conformally coupled scalar in AdS, independent of what values of the parameters M and ξ we started with in dS! The corresponding UV dimension of the dual operator is

$$\Delta_O = \begin{cases} \frac{d}{2} \\ \frac{d}{2} - 1 \end{cases}. \tag{3.10}$$

This ensures that the $< OO >$ two point function for the second choice reduces to the usual $\frac{1}{|x|^{d-2}}$ behavior of a scalar field in d dimensions.

3.2.3 Gravity

Let us repeat the same analysis for gravity. We start with the Einstein-Hilbert action in dS

$$S = M_d^{d-2} \int d^d x \sqrt{-g}(R - 2\Lambda) \quad (3.11)$$

where $\Lambda = \frac{1}{2}(d-1)(d-2)$. From table (1) we see that similar to the bulk scalar mass M the original dS cosmological constant Λ scales away. A big difference to the scalar field case this time is, however, that the gravitational kinetic term does *not* reproduce itself, instead we get a new gravitational action in AdS

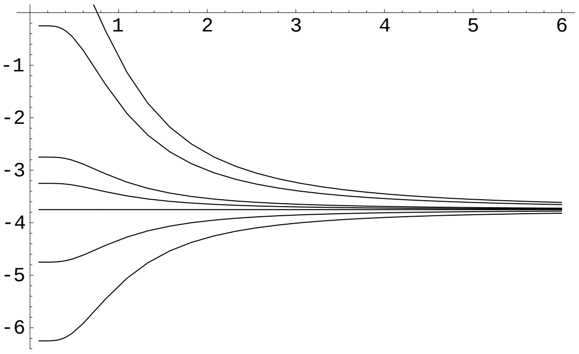

Figure 2: The position dependent AdS masses in $d = 5$ for various values of M and ξ.

$$\begin{aligned} S &= \int d^d x (M_d f)^{d-2} \sqrt{-g} \left(R - 2(d-1)(\nabla^2 \omega) - (d-2)(d-1)(\nabla\omega)^2 - 2f^2\Lambda \right) \\ &=_{ibp} \int d^d x \sqrt{-g} (M_d f)^{d-2} \left(R + (d-2)(d-1)(\nabla\omega)^2 - 2f^2\Lambda \right). \end{aligned} \quad (3.12)$$

We see that the dS/dS graviton corresponds to a varying Newton's constant in AdS, $M_d(z) = f(z) M_d$. As we approach the boundary the gravitational coupling increases. This allows for the localized graviton. The cosmological term is no longer constant, but involves covariant derivatives of ω. We will study solutions to the full non-linear equations that follow from (3.12) momentarily. To find the dimension of the operator dual to the graviton let us first consider linearized fluctuations. One expects the transverse traceless fluctuations of the graviton to satisfy the same equations as a massless scalar field. To read off the behavior of the dual FT operator, we only need the small z properties, where $\tanh(z) \sim z$. So our graviton with position dependent Planck mass should be equivalent to studying an auxiliary action

$$S = \int d^d x z^{d-2} \sqrt{-g} \partial_\mu \Phi \partial^\mu \Phi \,. \quad (3.13)$$

The equations of motion are

$$\frac{1}{\sqrt{-g}} z^{2-d} \partial_\mu z^{d-2} \sqrt{-g} g^{\mu\nu} \partial_\nu \Phi = \partial_\mu \partial^\mu \Phi = 0 \,. \quad (3.14)$$

Close to the boundary the graviton looks like a flat space graviton! In particular, the possible boundary behaviors are z^0 and z^1, as opposed to z^0 and z^4 for ordinary AdS gravity. This seems to yield two possibilities of the dimension of the dual operator, $d-1$ and $d-2$. Of course we expect the graviton to couple to the energy-momentum tensor with dimension $d-1$.

4. The UV structure of the holographic dual

4.1 Finite temperature

4.1.1 Equations of Motion

The most pressing question about the UV behavior of the dual theory is to what extent it is described by a local field theory coupled to gravity. One possible approach is to study the system at finite temperature. In particular the heat capacity of the system tells us how the degrees of freedom grow at high energies. A vanishing heat capacity, like found for the non-extremal NS5 branes, signals that the dual theory, in that case a little string theory, would certainly not be a local QFT. Alternatively, in even dimension one can calculate the Weyl anomaly following [15], which we will do in the next section. For both approaches one needs to construct solutions to the bulk equations of motion. In order to find those we need to first derive the equations of motions that follow from the action (3.12):

$$\begin{aligned} 0 = &\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) - f^{2-d}\left\{\nabla_\mu\nabla_\nu f^{d-2} - g_{\mu\nu}\nabla^2 f^{d-2}\right\} + \\ &+(d-2)(d-1)\left\{(\nabla_\mu\omega)(\nabla_\nu\omega) - \frac{1}{2}g_{\mu\nu}(\nabla\omega)^2\right\} + g_{\mu\nu}\Lambda f^2 \, . \end{aligned} \tag{4.1}$$

As a first check it is straight forward to verify that for our choice of f and Λ, $f = \tanh(z)$ and $\Lambda = \frac{1}{2}(d-1)(d-2)$, AdS_d written in the coordinate system (3.5) indeed solves the equations of motion (4.1).

4.1.2 Black Brane Solutions

In order to study the thermodynamic properties of the theory we would like to find the black brane solutions to (4.1). To map out the UV properties it is sufficient to study the somewhat simpler system with $f = z$ and $\Lambda = 0$. What this corresponds to is the conformal map of the $z > 0$ half of Minkowski space M_d to AdS_d,

$$ds^2_{AdS_d} = \frac{1}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right) = \frac{1}{z^2}ds^2_{M_d} \, . \tag{4.2}$$

For small z, $\tanh(z) \sim z$; in addition as $z \to 0$ the curvature on the dS_{d-1} slice can be neglected. So in the UV the thermodynamic properties of dS/dS can be studied by looking for black brane solutions of (4.1) with $f = z$ and $\Lambda = 0$.

After changing coordinates to $r = 1/z$, it is again straight forward to check that AdS_d written as

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{h(r)} + r^2 d\vec{x}^2 \tag{4.3}$$

with

$$h(r) = r^2 \tag{4.4}$$

solves the equations of motion (4.1). With the same ansatz (4.3) the most general solution for $h(r)$ is

$$h(r) = r^2 - \mu r \tag{4.5}$$

which describes a black brane in asymptotically AdS space with a horizon at

$$r_H = \mu \,. \tag{4.6}$$

The temperature of this black brane can be obtained by Wick rotating to euclidean space. To avoid a conical singularity, t_E must have periodicity

$$\beta = \frac{1}{T} = \frac{4\pi}{\mu} \,. \tag{4.7}$$

To determine the free energy F we can calculate the difference of euclidean on-shell gravitational actions

$$I = \beta F = -\frac{1}{16\pi G_N} \int d^d x \sqrt{g} f(z)^{d-2} \left(R + (d-2)(d-1)(\nabla\omega)^2 - 2f^2\Lambda \right) + I_{\rm GH} \tag{4.8}$$

between our solution and thermally compactified AdS as the reference spacetime with $F = 0$. In the usual Hawking Page analysis [16, 17] the on-shell value of the integrand is $-2(d-1)\sqrt{g}$ so that $16\pi G_N I$ just becomes $2(d-1)$ times the proper volume of spacetime. Also the Gibbons Hawking term I_{GH} in that case does not contribute, since the term in $h(r)$ proportional to μ falls of to rapidly with r. Plugging back (4.5) into the action we find that in our case the integrand is just $-2(d-1)$, since the $\sqrt{g}$ factor cancels against the position dependence of G_N. So instead of calculating the volume, in our case the action just measures the coordinate volume. Matching the periodicity β' of thermal AdS to the period of t_E at a fixed, large value R,

$$\beta' R = \beta\sqrt{R^2 - \mu R} \qquad \Rightarrow \qquad \beta' \approx \beta \left(1 - \frac{1}{2}\frac{\mu}{R} \right), \tag{4.9}$$

and compactifying the $\vec{x}$ coordinates on a volume V we find the bulk contribution to the free energy to be

$$16\pi G_N \beta F_{\rm bulk} = 2(d-1)V \left(\beta(R - r_h) - \beta' R \right) = -(d-1)V\beta r_H = -4\pi(d-1)V \,. \tag{4.10}$$

In addition we also have a contribution from the Gibbons Hawking term. For a metric of the form (4.3) the trace of the extrinsic curvature Θ_{ab} of a slice at $r = R$ is

$$\Theta(r = R) = -h^{1/2} \left(\frac{h'}{2h} + \frac{d-2}{r} \right). \tag{4.11}$$

With this we get for the black brane or thermal AdS background

$$16\pi G_N I_{\rm GH} = f(R)^{d-2} \int d^{d-1}x \sqrt{g_I}\Theta = \int d^{d-1}x \left((2d-3)\mu - 2(d-1)R \right) \tag{4.12}$$

where again the position dependence of Newton's constant with $f(r) = 1/r$ cancelled against the one in the square root of the determinant of the induced metric $\sqrt{g_I}$. Finally we get for the action difference

$$16\pi G_N \beta F_{\rm GH} = V\beta \left((2d-3)\mu - 2(d-1)R \right) + V\beta' 2(d-1)R = 4\pi(d-2)V \,. \tag{4.13}$$

Putting bulk and GH contribution together we finally obtain:

$$\beta F = -\frac{V}{4G_N}\,. \tag{4.14}$$

F is negative, so for all values of μ the black brane is thermodynamically preferred over thermal AdS. However, βF is independent of β, so the internal energy $E = \frac{\partial I}{\partial \beta}$ vanishes, and so does the heat capacity. The system has basically no degrees of freedom, the internal energy is independent of the temperature! Last but not least, one can calculate the entropy

$$S = \beta E - I = -I = \frac{V}{4G_N}\,, \tag{4.15}$$

which is precisely the usual quarter of the horizon area Vr^{d-2} in units of the local Newton's constant, $G_N(r) = G_N r^{d-2}$. While the analysis of the black brane for the $f = z$ theory shows that the heat capacity of the dS/dS system is approaching zero in the far UV, in order to determine the approach to zero one would have to do a similar analysis for the black branes in the $f = \tanh(z)$ theory. This is beyond the scope of the current analysis.

To close this section let us try to understand the physics of the black branes we found by undoing the conformal map that took us from Minkowski space to AdS. Instead of being solutions to an unconventional gravitational action in an AdS background they should then correspond simply to solutions of vacuum Einstein gravity. The black brane metric is conformal to

$$ds^2 = -\frac{h(r)}{r^2}dt^2 + \frac{dr^2}{r^2 h(r)} + d\vec{x}^2 \tag{4.16}$$

which in terms of

$$\rho = \frac{2(r-\mu)}{\mu\sqrt{h(r)}} \tag{4.17}$$

reads

$$ds^2 = -\frac{\mu^2\rho^2}{4}dt^2 + d\rho^2 + d\vec{x}^2\,. \tag{4.18}$$

In the original conformal frame all black brane solutions are just Rindler spaces with different time coordinates! Many of the thermodynamic properties of the black branes follow directly from this identification, but their interpretation in terms of a dual CFT required us to transform to AdS.

4.2 Conformal Anomaly

AdS/CFT instructs us to evaluate the bulk action on a given solution in order to calculate the boundary partition function for a given boundary metric. This quantity has divergences due to the infinite volume of AdS. In a coordinate system where the boundary sits at $z = 0$ the divergences will typically go like powers of z. These divergences map one-to-one to UV divergences in the field theory and can be dealt with in the same way: by adding local counterterms on the $z = \epsilon$ slice before taking $\epsilon \to 0$. In order to preserve diffeomorphism invariance these counterterms should be constructed from curvature invariants made out of the induced metric on the slice. For details on the procedure see [15]. However in even boundary dimensions (odd bulk dimension d) there are in addition $\log(z)$ terms and they represent the conformal anomaly.

For the standard Einstein Hilbert action on the dS_{d-1} sliced AdS_d background (3.5) the on-shell value of the integrand is $-2(d-1)\sqrt{g}$, as reviewed above in the black brane context. Hence

$$S_{\text{on-shell}} = \frac{-2(d-1)}{16\pi G_N} \int \frac{dz}{\sinh^d(z)} \sim \cosh(z)\, {}_2F_1\left(\frac{1}{2}, \frac{d+1}{2}, \frac{3}{2}, \cosh^2(z)\right). \tag{4.19}$$

Expanding the rhs in powers of z around $z=0$ one finds that in odd dimension the only divergent terms that appear are powers of z, while in even dimension there is indeed in addition a logarithmic term. In particular in $d = 3,\ 5,\ 7$ we obtain

$$\int \frac{dz}{\sinh^3(z)} = -\frac{1}{2z^2} - \frac{\log(z)}{2} + \mathcal{O}(z^0) \tag{4.20}$$

$$\int \frac{dz}{\sinh^5(z)} = -\frac{1}{4z^4} + \frac{5}{12z^2} + \frac{3}{8}\log(z) + \mathcal{O}(z^0) \tag{4.21}$$

$$\int \frac{dz}{\sinh^7(z)} = -\frac{1}{6z^6} + \frac{7}{24z^2} - \frac{259}{720z^2} - \frac{5}{16}\log(z) + \mathcal{O}(z^0)\,. \tag{4.22}$$

The log terms give the conformal anomaly evaluated on dS_2, dS_4 and dS_6 respectively. In the conventions of [15] we want to write the anomaly as

$$\mathcal{A} = -\frac{2}{16\pi G_N} a_{d-1}. \tag{4.23}$$

where $-\frac{2}{16\pi G_N} a_{d-1}$ is the coefficient of the $\log(z)$ term in the on-shell action[2]. So we get for dS_{d-1} backgrounds

$$a_2 = 1 \tag{4.24}$$

$$a_4 = -\frac{3}{2} \tag{4.25}$$

$$a_6 = \frac{15}{4}\,. \tag{4.26}$$

In 2d this uniquely fixes the anomaly.

$$\mathcal{A} = -\frac{c}{24\pi} R \tag{4.27}$$

and one can just read off $c = \frac{3}{2G_N}$. In 4d the conformal anomaly is given in terms of two numbers, the coefficient a of $C^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + R^2/3$ and the coefficient $-c$ of $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$. In order to determine both a and c we need to know the bulk solution for one more boundary metric. A trivial extension is to consider the "black-string" type metric, where the dS_{d-1} space on the slice gets replaced with the dS_{d-1} Schwarzschild black hole. It is well known that this metric solves the equation of motions with the same

[2]The extra factor of 2 comes about from the difference between our z and their ρ coordinate. What is important is how the action transforms under a conformal transformation and that is captured by $\mathcal{A}$. We put the factor of 2 into the definition of a_{d-1} in order to agree with [15] already on the a's

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value of the on-shell action, since the d-dim R only depends on the $(d-1)$-dim $R_{\mu\nu}$ on the slice, not on $R_{\mu\nu\rho\sigma}$. From this we immediately learn that $a = c$, since the Riemann^2 terms have to cancel from the anomaly. With this choice our value for a_4 agrees perfectly with the one found by [15] when evaluated on a dS or SdS background. In 6d three tensor structures show up and our crude methods of just plugging in two backgrounds for which we happen to know the full solution into the action only give us two linear combinations of those. Still, our value of a_6 is again in perfect agreement with [15] evaluated on dS_6.

Now let us repeat the same exercise for the gravitational action (3.12) with position dependent G_N. The position dependence of M_d is $M_d(z) = \tanh(z) M_d$ which gives an extra factor of $\tanh^{d-2}(z)$. Up to terms that remain finite as $z \to 0$

$$S_{\mathrm{on-shell}} = \int \tanh^{d-2}(z) \frac{dz}{\sinh^d(z)} = \int \frac{1}{\sinh^2(z)} \frac{dz}{\cosh^d(z)} = \frac{-1}{z} + \mathcal{O}(z) \,. \tag{4.28}$$

For all d the only divergent term is a universal $-\frac{1}{z}$ and there are no logarithms. The conformal anomaly vanishes.[3]

One possible interpretation is that lower dimensional gravity screens the central charge to be zero, just like is well known from 2d gravity on string theory worldsheets. In this scenario one does not even need a conformal field theory beyond scales $1/L$ since the gravitational dressing will also make any FT a CFT.

In order to explore this a little further let us use the same methods to evaluate the conformal anomaly for a smooth RS type domain wall as constructed in [20] with dS slices. The metric can be written in the form

$$ds^2 = e^{2A(z)} \left(ds^2_{dS_{d-1}} + dz^2 \right) . \tag{4.29}$$

and so the conformal map can be performed just as in the dS_d case with $f = \frac{\sinh(z)}{e^A}$ instead of $f = \tanh(z)$. A graviton localizes at the center of the brane where $A' = 0$. We can call this place $z = 0$. As long as $e^A(0)$ is finite, the gravitational part of the on-shell action still evaluates to $\frac{-1}{z} + \mathcal{O}(z)$. The only part of f^{d-2} that was essential in this result was that it vanishes as z^{d-2} for $z \to 0$ in order to lower the degree of divergence. The $\cosh(z)$ only matters when looking at the finite terms. To evaluate the full conformal anomaly, one needs to calculate the on-shell action for all fields. The fat walls of [20] are built from bulk scalars. We already worked out the conformal transformation properties of (3.6) before. While the scalar potential terms become irrelevant once transformed to AdS, the kinetic term remains unchanged. So we get a contribution to the on-shell action

$$S^{\mathrm{scalar}}_{\mathrm{on-shell}} = \int dz \sqrt{-g} (\Phi')^2 = \int dz \frac{(\Phi')^2}{\sinh^d(z)} \,. \tag{4.30}$$

After integration by parts and use of the equations of motion one sees that the the bulk contribution to the conformal anomaly is proportional to $\Box_{d-1} \Phi$ and hence vanishes for field configuration that are constant on the slice, as we want in order to preserve the

[3] A similar analysis has been performed in the context of the dS/CFT correspondence of [18] in [19]. In that case the putative dual does not involve gravity and the conformal anomaly doesn't vanish.

symmetry on the slice. The boundary terms one generates when integrating by parts do not vanish, in fact they are singular. However since they only involve powers of the sinh and not its integral, all divergences are power divergences that can be cancelled by local counterterms. No log terms arise. This result was first obtained in [21] who analyzed the conformal anomalies in general backgrounds with scalars turned on. This supports our conclusion that the zero central charge is due to gravitational dressing cancelling the matter central charge.

In the same spirit the universal UV dimension of the scalar fields can be understood as gravitational dressing. Naively one would think that gravitational dressing should bring the operator dimension to $d-1$ so that one can add it to the action. But the $(d-1)/2 \pm 1/2$ we find is consistent as long as we only add products of the form $O^1_{d/2} O^2_{d/2-1}$ to the action,[4] where the subscripts label the dimension of the operator and the superscripts the two CFTs. We know that the coupling of the two CFTs has to be achieved via its boundary interactions. Continuity across the UV brane in the original dS space means that the value of the field at the boundary in one AdS (dual to CFT 1) appears as a boundary condition in the second AdS (dual to CFT 2). The discussion of multi-trace operators in [22, 23] uses precisely the product operator $\mathcal{O} = O^1_{d/2} O^2_{d/2-1}$ in order to achieve boundary conditions of the type we want, at least in a folded version of our duality: instead of one scalar field living in 2 copies of AdS there one has 2 decoupled scalar fields in one copy of AdS. Since we are dealing with gravity in addition to scalar fields, for us the 2 copies of AdS are more appropriate in order to avoid having 2 gravitons living in the same space. The effect of adding $\mathcal{O}$ to the action is to set the leading and the subleading behavior of the scalar field on one AdS equal to the leading and subleading term in the other AdS respectively.[5]

5. The Black Hole/Black Hole correspondence

For further support for our picture of the FT that governs the scales $1/L < E < M_d$ we turn to the study of brane world black holes. According to [25], the classical solution describing a brane world black hole from the bulk point of view encodes the quantum corrections to the lower dimensional black hole solution due to the (C)FT.

Let us briefly review the arguments of [25] in the dS/dS context. For Hawking radiation to not backreact, one needs the Hawking temperature to be much less than the black hole mass M:

$$T \sim \frac{1}{r_H} << M\,, \qquad \text{where} \quad r_H^{d-3} = \frac{M}{M_d^{d-2}}\,. \tag{5.1}$$

[4] We note also that in our reparametrization of dS slicing of dS space the symmetry which is manifest is $SO(1,d-1) \times Z_2$ where the Z_2 factor exchanges two CFTs. Therefore we should add the same operator to the action with 1 and 2 interchanged.

[5] As discussed in [24], the addition of a single product of operators would lead to extreme nonlocality in the *internal* dimensions X of the $AdS \times X$ solutions in AdS/CFT because each operator corresponds to a particular Kaluza-Klein mode in the internal space and hence each factor in the product involves an integral over the internal space. In order to avoid that here, we can add an appropriate infinite linear combination of product operators corresponding to a local boundary condition in all dimensions.

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Hence one need $M >> M_d$. The black hole has to be much heavier than Planck scale. Since $M_d << M_{d-1}$, there is an interesting window

$$M_{d-1} >> M >> M_d \tag{5.2}$$

where the classical d-dimensional bulk solution remains uncorrected, while the brane world black hole has sub-planckian mass and hence is dominated by the backreaction of its own Hawking radiation. By constructing the classical bulk solution we therefore get information on the one point function of the stress energy tensor of the (C)FT. In particular, in the absence of a conformal anomaly, which we have shown to vanish for the dS/dS system in the last section, the Hawking radiation gives rise to a traceless energy momentum tensor.

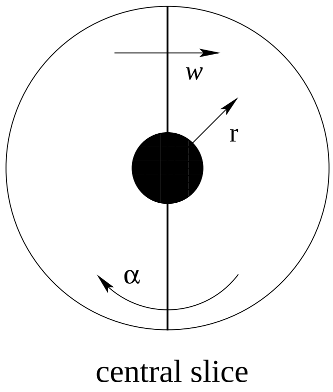


Figure 3: The bulk black hole appears as a quantum corrected brane black hole.

While for the dS/dS correspondence it did not matter which observer and hence which dS_{d-1} slicing of dS_d to pick — they are all related by dS_d transformations — for the black hole it is important. One choice is to study a black hole "behind" the cosmological horizon. In this case we expect the CFT to have a finite temperature which gets quickly diluted due to the dS expansion. For our purposes the more interesting slicing is the one where we take the bulk black hole intersecting the built in UV brane. In this case the standard Schwarzschild dS (SdS) bulk black hole becomes the quantum corrected brane world black hole.

To study its properties, let's recall some of the basic aspects of the dS/dS correspondence [1]. For pure dS we were using two coordinate systems:

$$ds^2 = -\cos^2(\theta)dt^2 + d\theta^2 + \sin^2(\theta)\left(d\alpha^2 + \sin^2(\alpha)d\Omega^2\right) \qquad \text{(static)} \tag{5.3}$$

and

$$ds^2 = \sin^2(w)\left(-\cos^2(\psi)dt^2 + d\psi^2 + \sin^2(\psi)d\Omega^2\right) + dw^2 \qquad \text{(dS sliced)}\,. \tag{5.4}$$

The change of variables is simply

$$\cos(\theta) = \sin(w)\cos(\psi) \tag{5.5}$$

$$\cos(\alpha) = \frac{\cos(w)}{\sin(\theta(w,\psi))}\,. \tag{5.6}$$

Now the bulk SdS black hole is easy to write down in the static slicing ($r = \sin(\theta)$)

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{h(r)} + r^2\left(d\alpha^2 + \sin^2(\alpha)d\Omega^2_{d-2}\right) \tag{5.7}$$

where

$$h(r) = 1 - r^2 - \frac{2\mu}{r^{d-3}}\,. \tag{5.8}$$

Even though finding the analog of the w-ψ slicing is difficult, it is easy to see that the $w = \pi/2$ slice (the built in UV brane) is still mapped to $\alpha = \pi/2$. This is fortunately all we need to know. So the metric on the brane is:

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{h(r)} + r^2 d\Omega_{d-3}^2 \,. \tag{5.9}$$

At first glance this looks like SdS on the brane again. But note that it has the wrong $h(r)$! This is still the $h(r)$ of d dimensions with its $\frac{2\mu}{r^{d-3}}$ instead of the $\frac{2\mu}{r^{d-4}}$ we would need in the appropriate $h(r)$ for a $(d-1)$ dimensional SdS black hole. Given the metric, one can work out what is the stress tensor that supports it, e.g for

$$\begin{aligned}
7 \to 6 : T^\mu_\nu &= \frac{5}{L^2}\mathrm{diag}(1,1,1,1,1,1) + \frac{\mu}{r^6}\,\mathrm{diag}(4,4,-2,-2,-2,-2) \qquad (5.10)\\
6 \to 5 : T^\mu_\nu &= \frac{4}{L^2}\mathrm{diag}(1,1,1,1,1) + \frac{\mu}{r^5}\,\mathrm{diag}(3,3,-2,-2,-2)\\
5 \to 4 : T^\mu_\nu &= \frac{3}{L^2}\,\mathrm{diag}(1,1,1,1) + \frac{\mu}{r^4}\,\mathrm{diag}(2,2,-2,-2)\\
4 \to 3 : T^\mu_\nu &= \frac{2}{L^2}\,\mathrm{diag}(1,1,1) + \frac{\mu}{r^3}\,\mathrm{diag}(1,1,-2)\\
d \to d-1 : T^\mu_\nu &= \frac{d-2}{L^2}\,\mathrm{diag}(1,\ldots,1) + \frac{\mu}{r^{d-1}}\,\mathrm{diag}(d-3,d-3,-2,\ldots,-2)\,.
\end{aligned}$$

The important conclusion here: T_{Hawking} is traceless! It is also straight forward to check that the $T^{\mu\nu}_{\text{Hawking}}$ we obtain this way is covariantly conserved, $\nabla_\mu T^{\mu\nu}_{\text{Hawking}} = 0$. Using Mathematica we have confirmed that $T_{\text{Hawking}} = 0$ holds as well in Kerr-de-Sitter backgrounds, as we have explicitly done for the cases of Kerr-dS-5 and Kerr-dS$_4$ [26]. They work the same way as their Schwarzschild cousins, on the central slice one gets something that almost looks like the lower dimensional Kerr black hole except for having the wrong power of r coming with the mass parameter in the metric function; the angular momentum part is completely standard.

6. Synthesis

Let us summarize our results for the regime $1/L < E < M_d$. Using the bulk description, which is still well approximated by classical gravity and field theory in this regime, we found that the holographic dual has effectively zero central charge (as evinced by vanishing conformal anomaly and asymptotic heat capacity) and scale dependent operator dimensions that in the UV tend to a universal value. The quantum stress tensor of the dual is traceless.

All these phenomena can still be interpreted as two conformal field theories coupled to gravity. As is well known from 2d gravity coupled to 2d CFTs, the dynamics of the conformal factor of the metric effectively cancels out any scale dependence of the matter sector and cancels the conformal anomaly due to the non-vanishing central charge of the matter sector. In addition, operators get gravitationally dressed since they are multiplied by powers of the scale factor. The dressed operators have a universal scaling dimension.

Continuity of the bulk fields requires that in addition we add direct double trace couplings between the two CFTs. The presence of some additional couplings between the two CFTs is also required for the tunneling transmission coefficient (3.3).

Although these results are qualitatively similar to those from solvable $2d$ gravity models, the details are different (for example the scaling dimensions of operators on each side are dressed not to dimension $d-1$ but to dimension $d/2, (d-2)/2$. It would be very interesting to interpret our results in detail using directly the holographic dual theory (2.1) including the interactions deduced from our computations here.

7. Changing Cosmological Constant and RG flow

The dS/dS correspondence shows that dS_d is holographically equivalent to a system which at low energies reduces to a pair of conformal field theories in $d-1$ dimensions, coupled to $d-1$ dimensional gravity.

In this section, we generalize this correspondence to situations like inflation and landscape decays where the cosmological term decreases in time. We find that this process in d dimensions is dual to a time dependent RG flow in $d-1$ dimensions, in which new light degrees of freedom come down as time evolves forward (realizing something similar to the idea in [4], but in a Lorentzian signature holographic correspondence). We also discuss the dual description of the formation of density perturbations, which involves the non-equilibrium statistical physics of the dual theory in the presence of the increasing number of light species.

The discussion in this section is qualitative, but this is sufficient to establish the effect of interest.

7.1 Inflation and RG

The dS/dS correspondence arises from foliating dS_d by dS_{d-1} slices

$$ds^2_{dS_d} = \frac{1}{\cosh^2\left(\frac{z}{L}\right)}\left(ds^2_{dS_{d-1}} + dz^2\right). \tag{7.1}$$

There is a convention in the description of the scales in this slicing, which is inconsequential in the case of a fixed cosmological constant [1] but which will play a role in our later discussion. Namely, we identified the curvature radius and inverse cutoff of the $d-1$ theory with the curvature radius L_d of dS_d. However, we could rescale the radius of the $d-1$ slices to some independent value L_{d-1}, as long as we maintain the relation that this radius scales like the inverse cutoff of the $d-1$ theory, $M_{UV} \sim 1/L_{d-1}$ in the sense discussed above. In this duality the number of effective species emerges as the Gibbons-Hawking entropy $N_{\rm species} \sim S = L^2 M_4^2$.

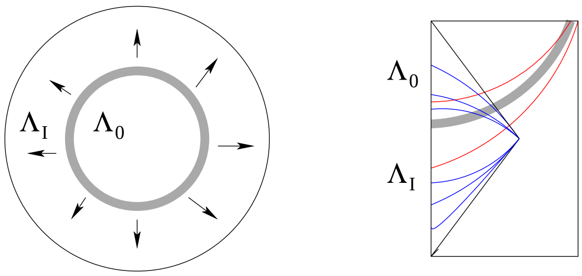
Now let us generalize this to inflation. In the 4-dimensional gravity theory, consider specifically a spatially flat slicing of the inflationary evolution

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \tag{7.2}$$

with $a(t) \sim e^{H_I t}$ for early times and $a(t) \sim e^{H_0 t}$ for late times, with $H_0 \ll H_I$. This is depicted in figure 4 (a), with constant t slices in grey, and corresponds to a tall Penrose diagram as in [27, 28]. The Hubble parameter varies with time t from an initial value H_I to a final value H_0.

If we project this onto the causal patch, as discussed in [27], we obtain a dynamics as depicted in figure 4 (b), with constant time slices for our $3d$ observer indicated in blue. Namely, the change in cosmological constant corresponds to a growing bubble of a phase H_0 inside a phase of H_I. The bubble wall may be thin or thick, depending on how fast the roll is from one de Sitter like phase to the other. A similar picture describes the result of non-perturbative bubble nucleation, in the case where the H_I phase is metastable.

As discussed above, in the dS/dS correspondence the number of species in each phase is $N_i \sim M_4^2/H_i^2$. The radial direction z corresponds to energy scale (with large $|z|$ closer to the horizon, infrared in the $3d$ description). Putting these features together with the inflationary dynamics, we see looking radially that the bulk of the inflationary dynamics corresponds to a time dependent RG flow in the 3-dimensional description. Namely, as we move from $z = 0$ to $z = \infty$, toward the infrared, we cross the bubble wall and the number of species decreases from N_0 to N_I. This describes a renormalization group flow in which the number of species decreases toward the IR, via running of masses and couplings in the dual description. In addition, these running masses and couplings are time dependent, in such a way that at fixed scale new light H_0 phase degrees of freedom are brought down as time evolves forward.



a b

Figure 4: Penrose diagram and spatial slice of inflationary spacetime in the causal patch.

It is also true that the transition appears as an expanding bubble in the $3d$ spacetime as well, though this feature would disappear in the reduction of the correspondence to the step $d = 2 \rightarrow d = 1$. Furthermore, we can now make use of the ambiguity discussed above in the scaling of the dS_{d-1} slices. Namely, by using the ambiguity in the scale L_3, we can maintain the same geometry on both sides of the bubble wall, thus relegating the entire effect of the transition to the question of the number of light species in the different phases. =-1Thus we arrive at the simple statement that inflation is holographically dual to a time dependent RG flow process in which new light species are brought down in time.

As discussed above, the original dS/dS correspondence is limited to a low energy effective field theory statement in $d-1$. The inflationary physics we are interested in here has largely to do with this regime, roughly because it is a theory of large scale physics with a resulting structure formation process which involves largely horizon physics. Once the bubble reaches the $|z_b| > L_0$ regime, it is well described by the low energy CFT in the $d-1$ description.

As in [29, 30], it would be interesting to go further and estimate the slow roll parameters in terms of the rate of RG flow in the system.

7.2 Reheating

The most naive form of reheating in the bulk involves the inflaton oscillating about the Λ_0 minimum and transferring its energy to bulk fields before settling down at the minimum.

In the $d-1$ description, this corresponds to a coupling oscillating in time about a new fixed point value and injecting energy into the CFT modes before settling down to the CFT fixed point value. The oscillation of the coupling changes the masses of the emerging CFT_0 degrees of freedom, which results in particle production generically. As in [31], as the coupling proceeds past the CFT value, the CFT_0 degrees of freedom mass up again and their energy density provides a restoring force trapping the coupling near its fixed point value. Hence inflation is dual to moduli trapping. This is especially precise if we make the choice of convention discussed above where we take the same geometry everywhere in the $3d$ spacetime, so that the entire effect is one of changing species number (time dependent RG flow).

7.3 Density perturbations

Inflation produces structure in the universe via a beautifully simple application of quantum field theory in the expanding universe. In the original flat spatial coordinates (7.2), the mode solutions to the equations of motion for the scalar perturbation grow in proper size in the $\vec{x}$ direction until they stretch outside the horizon of the approximate de Sitter of Hubble constant H_I, at which point their amplitude becomes constant. After the exit from inflation, the transition to H_0, these modes reenter the horizon and become dynamical again. The two point Greens function of these modes yields the Gaussian power spectrum seen in the CMBR observations.

It is interesting to ask how this process appears from our holographic screen. The initial dynamics of modes stretching toward the horizon takes forever from the point of view of the approximately static observer in the initial inflationary phase, if we approximate this as a de Sitter phase. In the radial direction, this stretching toward the horizon corresponds to a thermalization in the dual CFT at low energies. The next step, the exit from inflation, entails a relatively sudden influx of new light species in the $d-1$ description. This means that an excitation which was nearly thermalized with respect to the N_I species of the inflationary phase, is suddenly far out of equilibrium with respect to the new light N_0 species that have appeared. This is the $d-1$ dimensional holographic description of the "reentry" of modes into the horizon in the d dimensional description.

8. Conclusions

In this paper we worked out some basic features of the UV regime of the dS/dS correspondence. de Sitter space is a particularly symmetric example of a Randall-Sundrum system with a similar holographic dual description at the level of effective field theory. Bulk gravity calculations allow one to probe some of the basic physics of induced gravity above the scale where gravitational self-interactions become strong.

We computed several quantities determining how the lower dimensional theory behaves at energies above the cutoff. This resulted in a holographic verification that the total central charge and heat capacity is zeroed out and that a simple asymptotic dressing of operator dimensions arises. These are both features familiar in 2d gravity plus matter systems. Direct couplings between the two CFTs are also required.

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The duality naturally extends to situations with changing cosmological constant, such as inflation and landscape decays. For these processes the $d-1$ dimensional holographic description involves a time dependent RG flow in which new degrees of freedom become light in the transition.

Repeated application of our duality allows one to go to sufficiently low dimensions, that is two or one, so that gravity becomes non-dynamical and its effects reduce to constraints. As long as the matter part shows no pathologies, and the results of our current investigation suggest that it does not, one has a well defined holographic dual in terms of a constrained quantum mechanical system. We briefly outlined how this then can be used to construct observables for quantum gravity on backgrounds with accelerated expansion.

The motivation for this dual formulation is ultimately to provide a framework for the physics of accelerated expansion in the real universe (both with respect to early inflation and late acceleration). Although at large radius and low energies the effective weakly coupled description remains the bulk d dimensional one, the description in terms of $d-1$ dimensional physics may shed light on the physics of inflation and dark energy.[6]

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[6] For example the considerations in section 7 might provide a framework in which to address questions about the relation of holography to CMBR measurements, questions explored for example in [32].

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